Performance of the Kalman Filter of the Global Positioning System Operational Control Segment During January - March 1997*

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Abstract

We characterize the errors in the broadcast estimates of the Global Positioning System's satellite ephemeris and clock states made by the Operational Control Segment's (OCS) Kalman filter over the period when the system ephemeris Q's were changed. This change was called the Ephemeris Enhancement Endeavor (EEE). We report on the accuracy, stability, and periodic variations of the GPS Kalman filter's estimates of satellite clocks and ephemerides, particularly how they changed with EEE. Root-sum-squared (RSS) ephemeris error variations decreased by a factor of 1.7. The amplitude of the 2 cycle/d component decreased by a factor of 2.2. While the 2 cycle/d periodic effects dropped from 0.65 to 0.49 of the RSS ephemeris error, they still seem to be a major component. Radial ephemeris errors appear to be negatively biased by about 1 m. This bias is unchanged by EEE. Error deviations are generally no more than SV clock stabilities — the Kalman filter Q's are generally good. Clock estimate errors are worse than ephemeris errors — there may still be some room for improvement. Broadcast ephemeris errors add diurnal variations to the GPS time offset from the U. S. Naval Observatory Master Clock (USNO MC). GPS - USNO MC variations for averaging times near 1 d are consistent with White PM, averaging down to 1.5 ns at 1 d with 48 points/d. GPS - USNO MC appears to have periodic variations with periodicity spread approximately over periods from 16 to 24 d and a total amplitude of up to 4.5 ns.

INTRODUCTION

This report characterizes the performance of the ephemeris and clock parameters as broadcast from the Global Positioning System satellite vehicles (SVs) [1] during the first three months of 1997. During this period a change was made to refine the way these parameters are estimated by the Operational Control Segment's

(OCS) Kalman filter. This change was called the Ephemeris Enhancement Endeavor (EEE). EEE changed the estimated stochastic noise levels for the variations in the satellite ephemerides, the so-called ephemeris Q's of the Kalman filter for each satellite. These changes were made in the operational filter for each satellite at a specific time. Hence, it is possible to study the steady-state performance of the broadcast parameters from each satellite before and after the EEE changes were introduced. The precise ephemeris of the National Imagery and Mapping Agency (NIMA) [2] was used as the reference. Besides studying the broadcast ephemeris errors we also characterized the steering of GPS time to the U. S. Naval Observatory Master Clock (USNO MC), and the broadcast estimates of SV clock against GPS time.

BROADCAST EPHEMERIS ERRORS

To study the broadcast ephemeris errors, we compared the broadcast ephemerides to the NIMA precise ephemerides. We obtained the broadcast ephemerides from the Crustal Dynamics Data Information System (CDDIS) web site [3,4]. This is a NASA data bank on the Internet which, among other things, serves as a global data center for the International GPS Service for Geodynamics (IGS). We obtained the precise ephemerides directly from NIMA. The precise ephemerides gives x-, y-, and z-coordinates for each SV every 15 minutes. We evaluated the broadcast ephemerides for these 15-minute points and subtracted the position vectors from the precise ephemerides. The results were the ephemeris error vectors. We then projected these errors into radial, in-track, and cross-track components. In this way we produced three time series for each SV for the 90 d period, 1 point every 15 minutes, representing the SV's ephemeris errors.

BLIND SEARCH FOR EEE CHANGES

We looked for a change in the amplitude of the 2 cycle/d Fourier component. To search for a change, we broke the 90 d, 1 point every 15 minutes data into 256-point segments. This created data sets 2 2/3 d long. We computed the 2 cycle/d Fourier component for each of these segments, and looked for a significant drop in value. In some satellites, there was a clear drop, while in others the change was less clear.

The ratio of the 2 cycle/d component to the RSS of all Fourier components seemed to have no significant drop in much of the data. This suggests that the EEE changes improved the stability of the ephemerides in general, though they did not particularly reduce the 2 cycle/d periodic effects. As we shall see in later results, these are still about 0.5 times the total ephemeris error. The results of the blind search generally agreed within a few days of the actual change. The cases where the agreement is less good are those for which the drop was less obvious.

ANALYSIS OF EEE CHANGES

Using the actual dates for the EEE changes, we broke the data into two segments for each satellite. We then computed an estimate of the spectrum and the time variance, TDEV [5,6], for each satellite and for each of the vector error components: radial, in- track, and cross-track. There are two topics of particular interest from these studies: the 2 cycle/d periodic variations and the TDEV variations at 1 d. We discuss the 2 cycle/d periodic variations and the TDEV variations in this section, leaving the discussion of 1 d performance to go along with the discussion of SV clock estimate errors.

The periodic variations of ephemeris error are summarized in Table I. We use here the accepted formula for projecting error vectors on the earth by weighting radial, in-track, and cross-track terms as follows [7,8]:

$$Projected\ Error = \sqrt{Radial^2 + \frac{In\text{-}Track^2 + Cross\text{-}Track^2}{49}}. \quad (1)$$

For each segment and for each SV we report the projected values for the 2 cycle/d Fourier amplitude, the RSS of all Fourier components, and the ¼ d TDEV value. The RSS of all Fourier components equals the standard deviation. A periodic term affects the TDEV values, providing a maximum at half the period. Hence, the 2 cycle/d component appears in TDEV at the ¼ d averaging time. We also give the ratios of the 2 cycle/d Fourier amplitudes to the RSS of all Fourier components in Table II. This allows us to quantify how the EEE changes affected the noise level of the ephemeris error as a whole versus the 2 cycle/d periodic term. The RMS of this ratio over all SV's yields 0.65 before the changes and 0.49 after. Although there is significant reduction in the dominance of this periodic effect, it is still a major source of error.

GPS TIME VERSUS USNO MC

Measurements of GPS time against USNO MC were provided by USNO. These data had no selective availability (SA) effects and, after January 28, were corrected for measured ionospheric delays. Each data point is the average of 13 minutes of measurements of GPS time against USNO MC using one satellite. We graphed received data (Figure 1a). We corrected the data for the ephemeris error, using the NIMA precise ephemeris as "truth" (Figure 1b). We used a low-pass filter to remove white phase modulation (Figure 1c).

For each of the three time series, we interpolated the data to an even spacing of 48 points per day, then computed TDEV to estimate the spectrum. The TDEV graphs are displayed in Figures 2a, 2b, and 2c. (GPS - USNO MC) variations for averaging times near 1 d are consistent with white PM, averaging down to 1.5 ns at 1 d with 48 points/d. (GPS - USNO MC) appears to have periodic variations with periodicity spread approximately over periods from 16 to 24 d and a total amplitude of up to 4.5 ns. This periodicity was best seen in the TDEV plots. The periodic terms are so spread out that the spectral estimate gives a poor indication of the individual periodic components. A modulating frequency f_m with amplitude A, small compared to the high-frequency cut-off of the measurement system, affects TDEV according to [5,6]

$$TDEV(\tau) = A\sqrt{\frac{8}{3}} \cdot \left|\frac{\sin^3(\pi\tau f_m)}{\pi\tau f_m}\right|. \quad (2)$$

TDEV(τ) reaches its first maximum when $\tau f_m = ½$, or $\tau = \tau_p$, where τ_p is the period. In that case, $TDEV_{max} \approx A \cdot 1.04$. Hence, $TDEV_{max}$ is approximately the amplitude of the modulating periodic effect. If we consider that the TDEV of 4.5 ns at 10 d is the RSS of an effect due to periodicity and white PM of about 0.5 ns, that implies an amplitude of about 4.5 ns.

BROADCAST CLOCK CORRECTION ERRORS

We estimated the clock correction errors for each SV. We obtained the values of (GPS - USNO MC) for each SV from the data that USNO provided. We removed the ephemeris errors from these data, giving an estimate of (GPS - USNO MC) via that satellite. We then subtracted our filtered estimate of (GPS - USNO MC) from these data. The residuals are an estimate of the error in the broadcast value used to correct that SV clock for GPS time. We computed TDEV and a spectral estimate for these data for each SV. Since the SV clock stability determines its predictability, the clock correction error should be no worse than the clock stability at 1 d. Similarly, since the clock is the reference used to determine the satellite ephemeris, the 1 d TDEV of the ephemeris error should be less than the clock stability. Table III gives the 1 d TDEV values for the SV clocks, the ephemerides, and the clock correction errors. The instabilities of the ephemeris errors are all smaller than the clock instabilities. Values where the clock correction error instabilities are larger than the SV clock instabilities are in bold.

Finally, we looked at the mean and standard deviation values for the error terms. We computed these for the projected ephemeris errors for each satellite for both segments 1 and 2. We also compute these values for the clock correction errors for each satellite for both segments 1 and 2. We found that the mean ephemeris error terms are always negative. This is because the radial errors all have a negative bias of order 1 m, both before and after EEE. The overall RMS of the standard deviations is 1.8 m before EEE and 1.1 m after EEE. The clock errors appear unbiased, and their RMS standard deviations appear unaffected by EEE.

CONCLUSIONS

EEE significantly reduced the GPS satellite broadcast ephemeris error deviations, reducing the RSS ephemeris errors by a factor of 1.7 and the amplitude of the 2 cycle/day Fourier component decreased by a factor of 2.2. The 2 cycle/d periodic component remains a major part of the error. Radial ephemeris errors appear to be negatively biased by about 1 m. The reason for this is unknown. The Kalman filter Q's are generally good, though there may still be some room for improvement in the clock Q's. Broadcast ephemeris errors add diurnal variations to (GPS - USNO MC). (GPS - USNO MC) variations for averaging times near 1 d are consistent with white PM, averaging down to 1.5 ns at 1 d with 48 points/d. (GPS - USNO MC) appears to have periodic variations spread over 16 to 24 d with a total amplitude of up to 4.5 ns.

REFERENCES

[1] ``Navstar GPS Space Segment/Navigation User Interfaces," ICD-GPS-200, Revision C, 15 Dec 1994, ARINC Research Corporation, 4055 Hancock Street, San Diego, CA 92110.

[2] Malys, S., and J. Slater, ``Maintenance and Enhancement of the World Geodetic System 1984," Proc. ION-GPS-94, Salt Lake City, Sept. 20-23, 1994, pp 17-24 (1994).

[3] Flow, Archiving, and Distribution of Global GPS Data and Products for the IGS and the Role of the Crustal Dynamics Data Information System (CDDIS), Proc. of the Workshop on Improving the DGPS Infrastructure for Earth and Atmospheric Science Applications, March 1996, available from the CDDIS Internet web site.

[4] The Internet address for the CDDIS is http://cddisa.gsfc.nasa.gov/cddis.html

[5] ANSI Telecommunication Standard T1.101.

[6] D.W. Allan, M.A. Weiss, J.L. Jespersen, "A Frequency-Domain View of Time-Domain

Characterization of Clocks and Time and Frequency Distribution Systems," Proc. 45th Frequency Control Symposium, 1991.

[7] W. Fees, M. Menn, and Zeitzew, "OCS Performance Analysis," Proc. of the 1996 Performance Analysis Working Group (PAWG), Peterson Air Force Base, Colorado.

[8] C. Chuck, S. McReynolds, T. Metzger, and J. Moore, "GPS MCS Kalman filter performance," Proc. of the 1996 Performance Analysis Working Group (PAWG), Peterson Air Force Base, Colorado.

Table I: Broadcast Ephemeris Error, Periodic Behavior from Spectrum and TDEV
values in meters, component projected toward earth

		Segment 1			Segment 2		
SV	PR	2 Cycle/d	Total FFT	TDEV (¼ d)	2 Cycle/d	Total FFT	TDEV (¼ d)
13	02	1.2	1.7	1.6	0.4	0.9	0.6
14	14	0.9	2.9	1.4	0.5	1.6	1.1
15	15	1.2	1.7	1.8	0.5	1.0	0.8
16	16	1.0	2.8	1.8	0.2	1.0	0.6
17	17	1.3	1.8	1.8	0.5	0.9	0.8
18	18	0.4	1.3	1.1	0.3	1.1	1.1
19	19	0.5	1.0	0.8	0.3	1.3	0.8
21	21	1.4	2.3	2.1	0.4	0.8	0.6
22	22	1.6	2.0	2.0	0.7	1.2	1.1
23	23	1.0	1.9	1.9	0.9	1.9	1.5
24	24	1.1	1.5	1.4	0.4	0.7	0.6
25	25	0.7	1.1	1.0	0.4	0.9	0.7
26	26	0.7	1.0	1.0	0.1	0.9	0.6
27	27	0.8	1.2	1.1	0.6	1.1	0.9
29	29	0.6	0.9	0.9	0.7	1.0	1.0
30	30	2.0	2.5	2.4	0.2	0.8	0.5
31	31	0.9	1.4	1.6	0.5	0.8	0.8
32	01	1.1	1.5	1.8	0.6	1.0	1.1
33	03	1.6	2.2	2.2	0.6	1.2	1.1
34	04	0.7	1.1	1.0	0.3	0.7	0.5
35	05	1.3	1.9	1.8	0.2	0.6	0.4
36	06	1.8	2.4	2.6	1.3	1.7	1.8
37	07	1.5	2.1	2.4	0.6	0.9	1.1
39	09	0.4	0.7	0.7	0.4	0.8	0.6
40	10	1.8	2.8	2.5	0.3	0.9	0.7
RSS		5.93	9.28	8.60	2.69	5.37	4.57

Table II

FFT Ratios: $\frac{\text{2 Cycle/d Amplitude}}{\text{RSS of all Spectral Components}}$

SVN	PRN	Segment 1 Ratios	Segment 2 Ratios
13	02	0.71	0.44
14	14	0.31	0.31
15	15	0.71	0.50
16	16	0.36	0.20
17	17	0.72	0.56
18	18	0.31	0.27
19	19	0.50	0.23
21	21	0.61	0.50
22	22	0.80	0.58
23	23	0.53	0.47
24	24	0.73	0.57
25	25	0.64	0.44
26	26	0.70	0.11
27	27	0.67	0.55
29	29	0.67	0.70
30	30	0.80	0.25
31	31	0.64	0.63
32	01	0.73	0.60
33	03	0.73	0.50
34	04	0.64	0.43
35	05	0.68	0.33
36	06	0.75	0.76
37	07	0.71	0.67
39	09	0.57	0.50
40	10	0.64	0.33
RMS over all SV's:		0.65	0.49

Table III
One Day TDEV values
(in meters)

			Segment 1		Segment 2	
SVN #	PRN #	SV Clock	Ephemeris Error	Clock Correction Error	Ephemeris Error	Clock Correction Error
13	02	1.2	0.1	1.0	0.3	1.0
14	14	1.8	0.9	1.8	0.7	1.6
15	15	1.9	0.1	1.2	0.3	1.1
16	16	1.1	0.9	**1.3**	0.7	1.0
17	17	1.3	0.1	1.4	0.2	0.7
18	18	1.7	0.5	1.3	0.7	1.2
19	19	1.5	0.3	0.6	0.6	0.6
21	21	1.4	0.1	1.3	0.2	1.3
22	22	1.6	0.2	1.7	0.3	1.2
23	23	1.7	0.3	0.9	0.4	1.2
24	24	3.0	0.1	0.6	0.2	0.9
25	25	0.9	0.1	**1.0**	0.3	**1.1**
26	26	0.9	0.1	0.9	0.2	0.9
27	27	2.3	0.1	1.0	0.3	1.0
29	29	0.9	0.2	**1.1**	0.3	0.4
30	30	1.3	0.1	1.0	0.3	0.8
31	31	1.4	0.1	0.9	0.1	1.1
32	01	1.2	0.1	1.1	0.2	1.2
33	03	0.8	0.1	**0.9**	0.3	**0.9**
34	04	0.7	0.1	**0.8**	0.3	**1.1**
35	05	1.1	0.1	1.1	0.2	0.9
36	06	1.1	0.2	1.0	0.3	**1.3**
37	07	0.8	0.1	**0.9**	0.2	0.5
39	09	1.0	0.1	0.8	0.2	**1.1**
40	10	1.0	0.3	**1.1**	0.2	**1.1**

Figure 2a

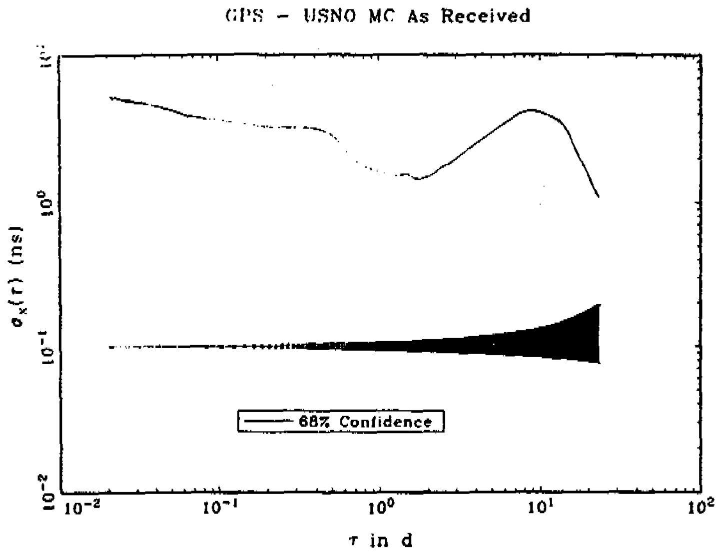

Figure 2b

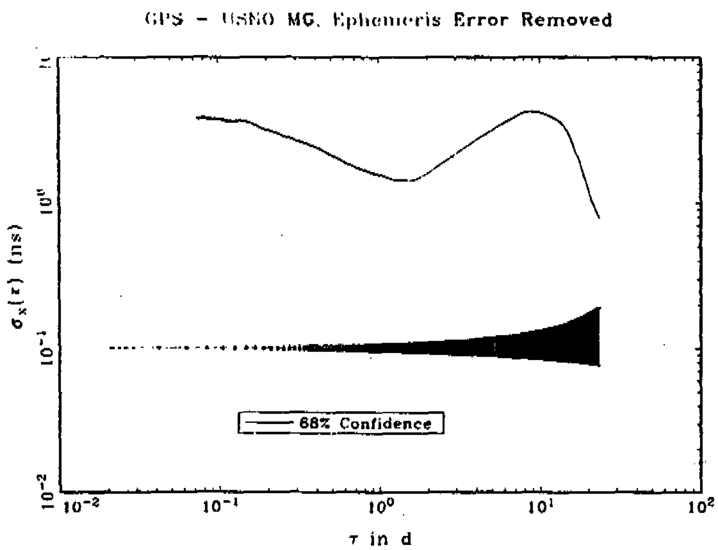

Figure 2c

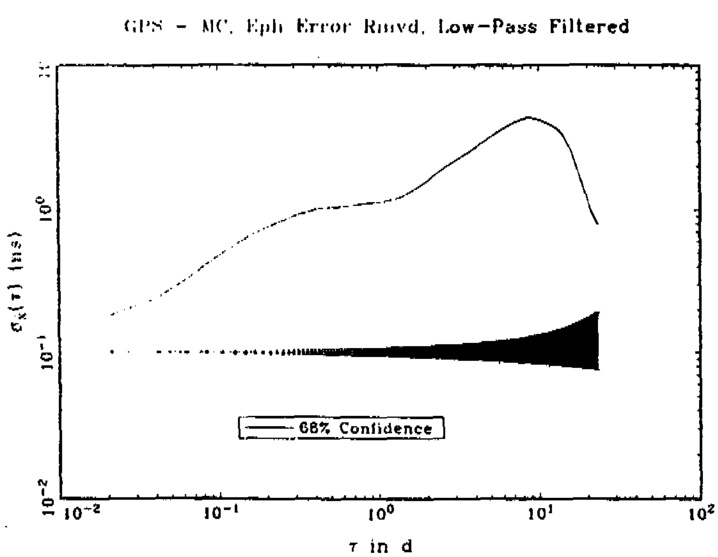

Figure 1a

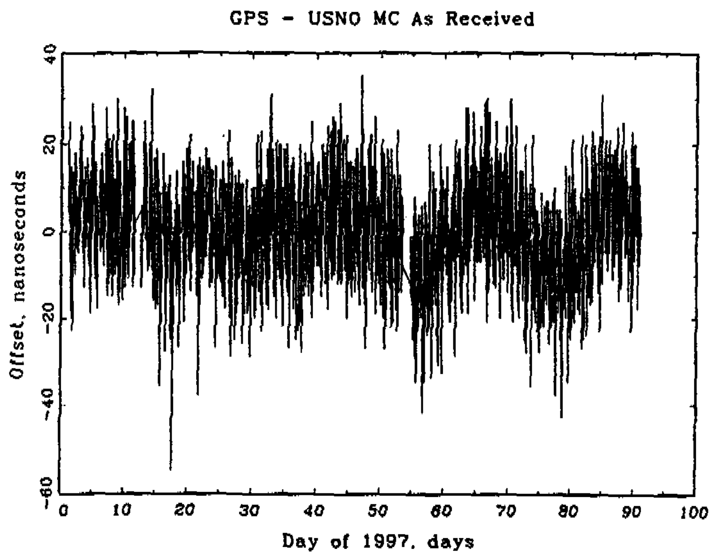

Figure 1b

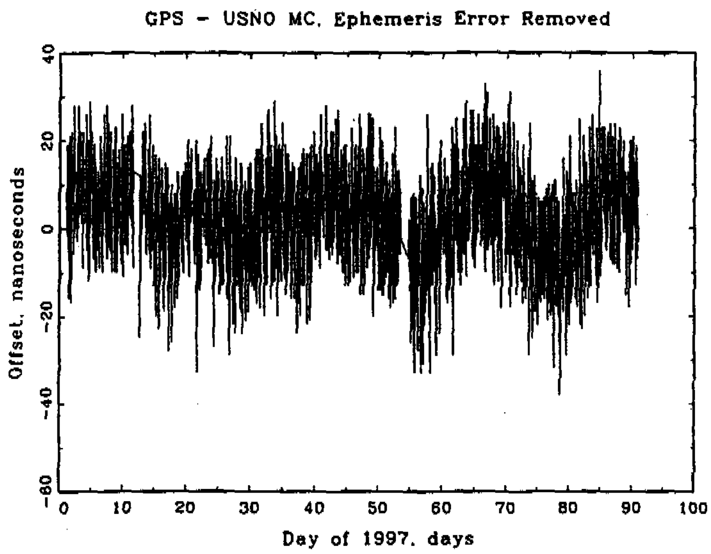

Figure 1c